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A MONTE CARLO EXPERIMENTAL INVESTIGATION OF THE EFFECT OF MULTIPLE-ITEM ORDERS ON INVENTORY COSTS

A THESIS

Presented to

The Faculty of the Graduate Division

by

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A MONTE CARLO EXPERIMENTAL INVESTIGATION OF THE EFFECT OF MULTIPLE-ITEM ORDERS ON INVENTORY COSTS

Approved: [illegible]

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Date approved by Chairman: 30 May 1963

DEDICATION

To my wife, Sara, whose encouragement and support have made this work possible.

FOREWORD

The subject of this thesis was first encountered in an industrial application. It was employed in a very sophisticated automated inventory system, but without genuine assurance of the benefits thought to accrue from it.

Discussion of the subject among colleagues prompted the view that little could be hoped for in the way of a closed-form solution, and that Monte Carlo techniques would offer greater potentials in testing relevant hypotheses.

Special thanks should be given to the Rich Electronic Computer Center for making this study possible through the availability of a large-scale computer.

Dr. Glenn Gilman, my thesis advisor, was most helpful and also added immeasurably to the manuscript. I am thankful for the criticisms, observations, and recommendations of Dr. P. B. Han and Dr. J. W. Walker, which improved the content and presentation significantly. I am especially grateful for the constructive and helpful attitude of my committee toward this endeavor.

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SUMMARY

An experimental investigation of alternative methods of replenishment ordering in an inventory system was made utilizing Monte Carlo models to simulate activity.

A basic system cost was developed by simulating a model in which the ordering system permitted only one item per order.

The concept of inventory families was developed, and a second model developed the system cost in which the ordering system permitted the combination of more than one item per order if, by sheer randomness, more than one item in the same family was at or below its order point.

A third model not only permitted stochastic combinations, but attempted to force more combinations by ordering an item, under certain conditions, before it reached its order point.

The concept of random combination proved to be an economic feasibility for large, active inventory families, and forced combination proved to be economically feasible for small, inactive inventory families. As a result of the findings of the present study, forced combination is recommended as a practical management technique by mutually exclusive utilization of it and the random combination technique applied to inventory families.

CHAPTER I

INTRODUCTION

Objective of Study

The objective of this study is to simulate the operation of several stochastic inventory models, using total cost analysis, for measuring the economic efficiency of certain ordering systems in inventory management.

Background of the Study

Inventory Activities

In general, an inventory system includes the following activities:

(1) Ordering items from suppliers.

(2) Receiving and inspecting items shipped from suppliers.

(3) Storing items for safety and protection until issue.

(4) Issuing items from storage for use.

(5) Periodically inspecting inventory levels to determine if a point has been reached at which another order should be initiated to replenish stock.

(6) Determination of the amount to order.

(7) Determination of the stock level at which to order.

The determination of how much to order and when to order are the key factors in inventory control for they determine the total costs of the system.

In recent years, much study has been given to inventory theory

and application. Such significant strides have been made in this area that a high degree of sophistication has developed in industrial applications of inventory control techniques. Although there are only a few basic inventory systems, there are many individual interpretations applied in industry which suit the needs of the individual firms.

Inventory System Applications

The common industrial inventory system is comprised of two principal functions: inventory accounting and inventory control.

Inventory Accounting. Inventory accounting has to do with the paper work of ordering, recording receipts, recording issues, and logging data--such as recording actual lead time elapsed, number of issues during the period, the number of stockouts, and other similar data. One very important part of this function is to watch the stock level of each item to see if it is time to place a replenishment order. If the stock level is at or below the order point, an order is initiated.

Inventory Control. Inventory control has to do with establishing the important parameters of the system which control inventory costs of the firm. As noted above, these control parameters are how much and when to order, and are referred to as the "order quantity" and "order point" respectively.

Order Quantity. The basic costs of an inventory system are ordering costs and keeping costs. The greater the quantity ordered at one time, the fewer times it is necessary to order in a given time period. Obviously, since fewer orders have to be written, order costs are lower. On the other hand, if large quantities are ordered, they must also be stored--thus increasing storage (or keeping) costs.

Then, at some point between minimum order costs and maximum keeping costs, and vice-versa, there is a cost which is minimum for the sum of the two. This point turns out to be where ordering costs and keeping costs are equal.

The order quantity then, is basically determined by buying that quantity which will equate ordering costs with keeping costs.

Order Point. The order point is set at a level that will achieve some desired result: e.g., to replenish stock after running out; get more stock before running out; or, to get more stock before running out during some predetermined percentage of the number of times that orders are placed.

These two parameters are usually determined periodically--once or twice a year, or quarterly--and they are used by the inventory accounting portion of the inventory system.

Supplier Selection. In practice, a firm may submit bids or otherwise select a supplier for a given item. Once the supplier is selected, all orders originating for a given item are sent to that supplier. What actually happens, in many cases, is that a supplier is selected, not for just one item (e.g., steel bolt; 1 in. by $\frac{1}{2}$ in.), but for a group of items (e.g., all steel bolts needed: $\frac{1}{2}$ in. by $\frac{1}{2}$ in.; 1 in. by $\frac{1}{2}$ in.; $1\frac{1}{4}$ in. by $\frac{1}{2}$ in.; etc.). This gives rise to the possibility of multiple-item orders.

Multiple-item Orders. Where there are a number of similar items which may differ only physically--bolts, pipe, steel wire, containers, electrical wire, nails, light bulbs, valves, wrenches, tires, spark plugs, conduits, lumber, gloves, vacuum tubes, photographic film, to name a few--

they may have the same delivery lead time, same cash terms, same shipping method, and, of course, the same supplier. This group of items, then, may be called a family of items.

Within these families, if there is more than one item which must be ordered (i.e., they are either at or below their order points), there are economic reasons why they should be placed on the same order: hence, a multiple-item order is defined as an inventory replenishment document which includes requests for more than one item, each of which is below its respective order point.

Optional-multiple-item Orders. An item may not be at or below its order point, but nevertheless so near that it would be economical to include it on a multiple-item-order by saving the outlay of fixed costs F associated with each order. The fixed costs would tend to be offset by increased keeping costs, but if the proper parameters were established there should be a net savings on the average.

The inclusion of such an item on an order will, then, be optional. Hence an optional-multiple-item order is a multiple-item order which includes at least one optional item.

Economics

Total costs are a function of ordering costs and keeping costs. Ordering costs are divided into two major components: fixed and variable. The fixed costs, F, are incurred in the administrative and technical process of placing the order--the cost of processing the piece of paper, so to speak. The variable costs, V, are incurred by the physical and administrative acts involved in receiving, inspecting, storing, and reporting data on the individual item.

Let item-order cost be the cost of ordering an individual item. Then, if an order is issued for a single item, the item-order costs, IOC is:

$$IOC = \frac{F + V}{1}$$

But an order for more than one item lowers the item-order cost because

$$IOC = \frac{F + NV}{N}$$

Where N is the number of items on the order, F is distributed over N items.

Literature Search

The literature is silent on the concept of optional-multiple-item ordering systems. As far as can be determined, the subject of the present study is novel.

Conclusion

There are potential economic gains to be realized in inventory systems by lowering ordering costs. This study is an experimental investigation of the costs of a basic inventory system to evaluate the hypotheses that:

(1) Multiple-item ordering systems decrease inventory costs as compared to single-item ordering systems.

(2) Optional-multiple-item ordering systems, a novel technique, decrease inventory costs as compared to multiple-item ordering systems.

CHAPTER II

EQUIPMENT AND FACILITIES

Rich Electronic Computer Center

The facilities of the Rich Electronic Computer Center, a Division of the Engineering Experiment Station, Georgia Institute of Technology, were made available for this study under the sponsorship of the Engineering Experiment Station.

The principal equipment which made this study possible is the Burroughs 220 computer used for simulating the three Monte Carlo inventory models. Four programs were used and are discussed in Chapter III, Development of Models.

The Burroughs 220 is a general-purpose, stored-program, automatic, sequentially-controlled, decimal, digital, computer system.

A 5000-word memory (10 digits plus sign), a 240-card/minute reader, six dual-lane magnetic tape units with a transfer rate of 25,000 characters/second, two line printers (900 lines/minute and 150 lines/minute), a high speed paper tape reader and paper tape punch, a 100-card/minute card punch, and a supervisory typewriter comprise the major elements of the Burroughs 220 computer system. The Computer Center also provided auxiliary equipment essential to the prosecution of this study such as card punches and a sorter. All programs were written in an algebraic-like compiler language which is the Burroughs' implementation of ALGOL.

CHAPTER III

DEVELOPMENT OF MODELS

Summary

There are three models required in this study. They are:

(1) The Monte Carlo Model for determining TVC with a single-item ordering system.

(2) The Monte Carlo Model for determining TVC with a multiple-item ordering system.

(3) The Monte Carlo Model for determining TVC with an optional-multiple-item ordering system.

The three models simulate activities on a sample inventory of items with given order points and order quantities.

Development of these four models include the development and implementation of algorithms for computer manipulation.

Discussion

The Sample Inventory

The sample inventory selected is comprised of 46 items: bolts, valves, and V-belts. The nature or characteristics of these families may be noted. Family I, bolts (items 101 through 131), contains 31 items which are characterized by high volume, low unit cost, and a 14-day lead time. Family II, valves (items 201 through 205), contains five items which are characterized by low volume, high unit cost, and a 30-day lead time. Family III, V-belts (items 301 through 310), contains 10 items of medium volume, medium cost, and an 8-day lead time. These

characteristics should aid in detecting influences on costs associated with the three ordering systems.

Order Quantities and Order Points. There are numerous methods for determining order quantities and order points. This present study does not address itself to either of these problems. However, since both of these parameters must be available to the simulation models, the following assumptions are made:

(1) Order quantities are given by minimizing

$$TVC = \frac{RS}{Q} + \frac{QCI}{2}$$

where: TVC = Total variable cost

R = Annual volume in units

S = Total costs of ordering on single-item orders

Q = Economic order quantity

C = Unit cost of an item

I = Inventory carrying rate

Differentiating:

$$\frac{dTVC}{dQ} = \frac{-RS}{Q^2} + \frac{CI}{2}$$

Setting equal to zero and solving for Q,

$$Q^2 = \frac{2RS}{CI}$$

$$Q = \sqrt{\frac{2RS}{CI}}$$

It is assumed that S = \$8.00 and I = 0.20 in the above equation.

(2) The order points were established under the assumption that an item would be in stock during at least 95 per cent of the order cycles. The conditional probability P of no shortage during a given lead time ℓ, for Poisson demand, is:

$$P\left[d \leq OP \mid \ell \right] = \left[\sum_{k=0}^{OP} \frac{e^{-\lambda\ell} (\lambda\ell)^k}{k!} \right] = 0.95$$

where: d = demand during lead time

ℓ = lead time

λ = rate of demand

These computations lend themselves to tediousness and error when performed manually. Because of this, a computer program was written which uses basic data on each item to calculate the order point and order quantity, and prepares an inventory file for subsequent use in the simulation programs.

Appendix A presents this computer program. The inventory data, including the parameters as they were prepared by the program, are presented in Table 1. The algorithm for this program is:

(1) Punch into cards data on each item to be included in the inventory.

(2) Mount a reel of blank magnetic tape on the computer on which the inventory records are to be copied.

(3) Read a data card into the memory of the computer.

(4) If all cards have been read, go to step 12.

TABLE 1. INVENTORY FILE AND CONTROL PARAMETERS

ITEM NO.	DESCRIPTION	ANNUAL USAGE	UNIT COST	NO. ISSUES	LEAD TIME	ORDER QTY	ORDER POINT
101	BOLT 3/4 X 1/4	23052	0.0225	52	14	9053	2215
102	BOLT 3/4 X 5/16	3796	0.0335	16	14	3010	474
103	BOLT 3/4 X 3/8	8544	0.0455	64	14	3875	665
104	BOLT 1 X 1/4	14568	0.0235	86	14	7042	1183
105	BOLT 1 X 5/16	5052	0.0345	39	14	3422	516
106	BOLT 1 X 3/8	5876	0.0470	67	14	3162	435
107	BOLT 1 X 7/16	7848	0.0790	36	14	2819	872
108	BOLT 1 X 1/2	2256	0.0915	29	14	1404	231
109	BOLT 1 X 5/8	2188	0.1585	14	14	1050	312
110	BOLT 1 X 3/4	1156	0.2250	18	14	641	128
111	BOLT 1-1/4 X 1/4	8756	0.0250	91	14	5293	672
112	BOLT 1-1/4X5/16	12160	0.0360	68	14	5198	1068
113	BOLT 1-1/4 X 3/8	15688	0.0495	96	14	5035	1141
114	BOLT 1-1/4X7/16	6472	0.0830	42	14	2497	616
115	BOLT 1-1/4 X 1/2	1248	0.0970	8	14	1014	156
116	BOLT 1-1/4 X 5/8	232	0.1630	12	14	337	38
117	BOLT 1-1/4 X 3/4	880	0.2250	8	14	559	110
118	BOLT 1-1/2 X 1/4	4152	0.0265	56	14	3540	370
119	BOLT 1-1/2X5/16	30616	0.0385	16	14	7976	3826
120	BOLT 1-1/2 X 3/8	5396	0.0525	73	14	2867	438
121	BOLT 1-1/2X7/16	36800	0.0870	79	14	5817	2790
122	BOLT 1-1/2 X 1/2	45768	0.1025	44	14	5976	4160
123	BOLT 1-1/2 X 5/8	1168	0.1760	7	14	728	166
124	BOLT 1-1/2 X 3/4	64	0.2380	2	14	146	32
125	BOLT 1-3/4 X 1/4	17316	0.0295	64	14	6852	1350
126	BOLT 1-3/4X5/16	5868	0.0430	62	14	3304	470
127	BOLT 1-3/4X 3/8	9280	0.0575	46	14	3593	804
128	BOLT 1-3/4X7/16	15564	0.0925	39	14	3668	1596
129	BOLT 1-3/4 X 1/2	19312	0.1095	65	14	3756	1485
130	BOLT 1-3/4 X 5/8	4824	0.1760	82	14	1480	348
131	BOLT 1-3/4 X 3/4	334	0.2380	6	14	335	55
201	VALVE 1-1/2	8	52.05	5	30	3	2
202	VALVE 2	3	63.30	3	30	1	1
203	VALVE 2-1/2	5	69.45	4	30	2	1
204	VALVE 3	4	82.65	2	30	1	2
205	VALVE 3-1/2	7	105.75	5	30	2	2
301	V-BELT 3/8 X 24	84	1.24	12	8	73	7
302	V-BELT 3/8 X 30	36	1.34	6	8	46	6
303	V-BELT 3/8 X 32	78	1.38	7	8	67	11
304	V-BELT 3/8 X 34	114	1.42	19	8	80	12
305	V-BELT 3/8 X 35	36	1.45	6	8	44	6
306	V-BELT 3/8 X 36	102	1.47	13	8	74	7
307	V-BELT 3/8 X 38	90	1.53	15	8	68	6
308	V-BELT 3/8 X 40	132	1.56	19	8	82	12
309	V-BELT 3/8 X 41	54	1.60	5	8	51	10
310	V-BELT 3/8 X 42	108	1.63	12	8	72	9

(5) Calculate the number of units required for protective stock.

(6) Record the quantity calculated in step 5 as the order point.

(7) Calculate the economic order quantity.

(8) Record the quantity calculated in step 7 as the order quantity.

(9) Write on magnetic tape all of the data taken from the data cards, and the parameters calculated in step 5 and step 7.

(10) Print out the same data as is written on tape in step 9.

(11) Go to step 3.

(12) Mark the end of the inventory file on magnetic tape.

(13) Rewind the tape for dismounting.

(14) Stop.

Monte Carlo Models

General

The Monte Carlo models simulate a basic inventory system described by T. L. Newberry (2) as the variable cycle inventory policy. The usual assumptions that withdrawals from inventory are for unity each time, that the stock level is under continuous surveillance, and that a replenishment order can be placed with a vendor at any time, are removed in the present study.

Instead, the stock level is inspected at the end of the daily activities, the orders are placed at specified intervals, the issue size is a random variate, and the lead time is a random variate. This inventory situation is illustrated in Figure 1.

There are three Monte Carlo models which simulate inventory activities for five-year periods. Each model begins simulating under identical initial inventory conditions, and the only difference between the models is the method of ordering replenishment stock.

The occurrence of a specific event is produced by the respective event generators:

(1) Issue time generator.

(2) Issue size generator.

(3) Lead time generator.

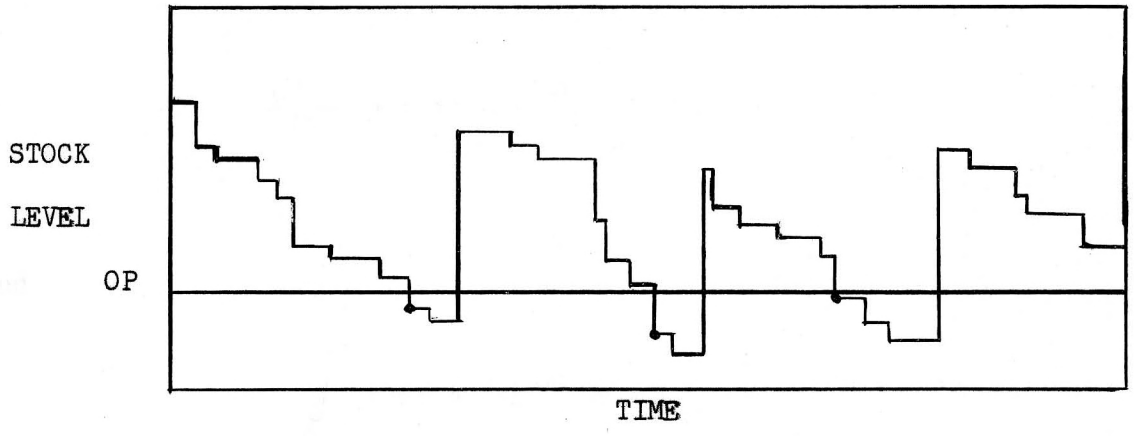
Random Number Generator. Each event generator is dependent on the availability of a random number. Pseudo-random numbers are computed by a method used by Teichroew (3) and others in the simulation of stochastic processes. The sequence

$$x_o = \text{any odd number, 10 digits, } x_{n+1} = 5^{13} x_n \pmod{10^{10}}$$

is easy to use, is fast, and is particularly suited to the Burroughs 220 computer word size and arithmetic logic circuitry.

Issue Time Generator. Issue events can be generated by specifying the time the event is to occur. If arrivals of demand for any item are given by the Poisson approximation

$$P(\alpha = k|t) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$



Actual order point represented by •

Figure 1. Illustration of the Inventory System

where λ = rate of arrivals, the probability of having at least one arrival during time t is

$$P(\alpha \geq 1|t) = \sum_{k=1}^{\infty} \frac{e^{-\lambda t}(\lambda t)^k}{k!} = 1 - e^{-\lambda t} = F(t)$$

a cumulative distribution function (see Figure 2).

It is well known that if arrivals are Poisson, then the time between arrivals has the exponential density (see Figure 3). That is

$$\frac{dF(t)}{dt} = f(t) = \lambda e^{-\lambda t}; \quad 0 < t < \infty$$

and

$$\int_0^{\infty} f(t)dt = \int_0^{\infty} \lambda e^{-\lambda t}\, dt = 1$$

Therefore, to simulate a random variable T with density f(t), let Y = F(T), where Y has a uniform density (i.e., be a random fraction).

$$Y = 1 - e^{-\lambda t}$$

$$e^{-\lambda t} = 1 - Y$$

$$T = \left| \frac{\ln(1-Y)}{\lambda} \right|$$

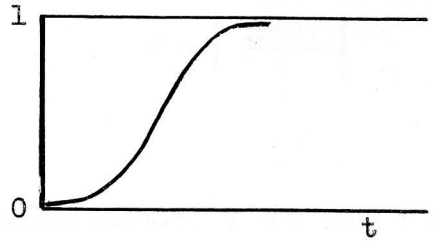


Figure 2. Cumulative Probability Distribution

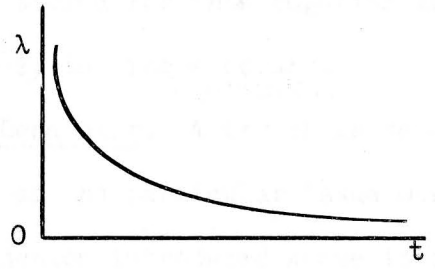


Figure 3. Exponential Density

Since 1-Y and Y are both uniform, the use of Y does not alter the results. If r = a random fraction, then

$$T = \left| \frac{\ln(r)}{\lambda} \right|$$

After each issue occurs, a new time T between issues is calculated. If $T \leq 1$, it is assumed that more than one issue has occurred on the current day d_c. If $T > 1$, the next issue is computed to occur on some date in the future d_f, where

$$d_f = d_c + T$$

The value of d_f is stored for interrogation as simulated time passes by. When d_f arrives, the issue occurs.

Issue Size Generator. After it is detected that an issue is to occur, the size of the particular issue must be determined. Again, the exponential function introduced above is followed, and is a practical method of generating issue sizes--especially where the mean issue size is large. Graves (1) notes its acceptance and the present writer endorses it because of its speed and utility. The size of the issue s is computed as follows: Let M = mean issue size in units, then

$$F(s) = 1 - e^{-\frac{1}{M} s}$$

It follows from the previous derivation that

$$\ln(r) = \frac{-1}{M} s$$

$$s = \left| M(\ln(r)) \right|$$

Lead Time Generator. Lead time L is computed by a technique similar to that used in computing time between issues. The nominal lead time ℓ is known and the length of the random lead time L is obtained by letting r = a random fraction, and evaluating

$$\ln(r) = \frac{-L}{\ell}$$

$$L = \left| \ell(\ln(r)) \right|$$

In order to better simulate a real situation, if

$$L < 2$$

it is recomputed until a computation yields

$$L \geq 2$$

arbitrarily assuming that it is more reasonable not to expect receipt of an item in less than two days from order initiation. This assumption is not deemed critical to any of the models presented in this present study.

General Assumptions. There are six general assumptions which are common to the three models, and they are stated here to avoid repetition.

(1) The basic inventory parameters have been established and are available to each model on magnetic tape.

(2) The issue γ of an item is a random event with respect to time.

(3) The size of an issue γ is a random variate.

(4) The lead time L is a random variate.

(5) The fixed ordering costs F and variable ordering costs V are known constants where F = \$5.00 and V = \$3.00.

(6) The mean values of the random variates have been recorded and are available to each model on magnetic tape.

Monte Carlo Model I

Model I develops costs of the basic system to which those of Models II and III are compared.

In addition to the general assumptions, Model I simulates inventory operations under the assumptions that:

(1) Only single-item orders are written.

(2) Orders are written daily as they occur, there being no practical value in doing otherwise.

This model is the single-item order model. The costs of each order are comprised of the fixed costs F and variable costs V; hence, the item order cost IOC = F + V in every instance.

The computer program listing is shown in Appendix B. The algorithm for this model is:

(1) Initialize random number generator.

(2) Read inventory records into memory from magnetic tape.

(3) Initialize inventory quantities by setting stock level of each item equal to its order quantity.

(4) Initialize the day counter.

(5) Set the item index to first item.

(6) If an order is due from the supplier for this item, go to step 8.

(7) Go to step 11.

(8) Increase the stock level by one order quantity.

(9) Remove the note from the record that an order is outstanding.

(10) Compute and record keeping costs of the receipt from the current day until the next issue.

(11) If this item is due to have an issue made from stock, go to step 13.

(12) Go to step 18.

(13) Determine the size of the present issue.

(14) Decrease the stock level by the amount determined in step 13.

(15) Determine when the next issue is to be.

(16) If there is an issue on the present day, go to step 13.

(17) Note in the item record the date of the next issue.

(18) Compute and record the keeping costs of the present inventory between the current date and the next issue.

(19) If the present stock level is at, or below, the order point, go to step 21.

(20) Go to step 24.

(21) Determine the length of the lead time to be experienced by the current order.

(22) Note in the item record that an order is outstanding and its due date.

(23) Compute and record the ordering costs for this item.

(24) If all items of the inventory have not been considered for the present day, go to step 27.

(25) Increase the day count by one.

(26) If the current day is the end of a year, go to step 29.

(27) Increase the item index to the next item.

(28) Go to step 6.

(29) Print values of annual keeping and ordering costs for each item.

(30) Print values of annual summary keeping and ordering costs for each family.

(31) If the current day is the end of the five-year period, go to step 33.

(32) Go to step 5.

(33) Stop.

Monte Carlo Model II

Model II is the multiple-item ordering model which permits items in the same family to be grouped into a single order if they are at, or below, their respective order points.

The additional assumptions employed in Model II are:

(1) Both single-item orders and multiple-item orders are permitted.

(2) Orders are written once each week.

(3) Family items are grouped into the same order for all

$$\Phi_i \leq OP_i$$

The second assumption allows a week of activity in between the order writing process which enhances the probabilities of larger multiple-item orders than the previous case where they were written daily. The reader may question the procedure of comparing the results of Model I and Model II because of this condition. However, it should be recalled that, under the strict single-item ordering system, there was no advantage for the weekly ordering rule to be operative.

Appendix C presents the computer program listing for Model II, and the algorithm is:

(1) Initialize the random number generator.

(2) Read inventory records into memory from magnetic tape.

(3) Initialize inventory quantities by setting stock level of each item equal to its order quantity.

(4) Initialize the day counter.

(5) Set the item index to first item.

(6) If an order is due from the supplier for this item, go to step 8.

(7) Go to step 11.

(8) Increase the stock level by one order quantity.

(9) Remove the note from the record that an order is outstanding.

(10) Compute and record keeping costs of the receipt from the current day until the next issue.

(11) If this item is due to have an issue made from stock, go to step 13.

(12) Go to step 18.

(13) Determine the size of the present issue.

(14) Decrease the stock level by the amount determined in step 13.

(15) Determine when the next issue is to be.

(16) Go to step 13 if there is to be an issue on the current day.

(17) Note in the item record, the date of the next issue.

(18) Compute and record the keeping costs of the present inventory between the current date and the next issue.

(19) If the present stock level is at or below the order point, go to step 21.

(20) Go to step 27.

(21) Determine the length of the lead time to be experienced by the current order.

(22) Note in the item record that an order is outstanding and its due date.

(23) If another item in the same family is already on order, denoted by the family Boolean digit, go to step 26.

(24) Set the family Boolean digit equal to one.

(25) Note in the item record that this item incurs the fixed costs associated with an order.

(26) Note in the item record that this item incurs the variable costs associated with an order.

(27) If all items of the inventory have not been considered for the current day, go to step 31.

(28) Increase the day count by one.

(29) If the current day is the end of a week, go to step 33.

(30) If the current day is the end of a year, go to step 36.

(31) Increase the item index to the next item.

(32) Go to step 6.

(33) Compute and record the ordering costs for all items.

(34) Set the family Boolean digit equal to zero for each family.

(35) Go to step 30.

(36) Print values of annual keeping and ordering costs for each item.

(37) Print values of annual summary of keeping and ordering costs for each family.

(38) If the current day is the end of the five-year period, go to step 40.

(39) Go to step 5.

(40) Stop.

Monte Carlo Model III

Model III presents a novel technique of including an item on an order (under certain conditions) even if the stock level is above the order point.

There are four additional assumptions which make this the optional-multiple-item ordering model:

(1) Orders are written once a week.

(2) Single-item, multiple-item, and optional-multiple-item orders are permitted.

(3) Family items are grouped into the same order for all

$$\Phi_i \leq OP_i$$

(4) Family items are grouped into the same order for all

$$\left(\frac{\Phi_i - OP_i}{2}\right) C_i I \leq U$$

where U is a constant.

These conditions will yield data for determining whether the hypothesis is true or false that ordering costs in Model III are lower than those in Model II.

The computer program listing is shown in Appendix D, and the algorithm for this model is:

(1) Initialize the random number generator.

(2) Read inventory records into memory from magnetic tape.

(3) Initialize inventory quantities by setting stock level of each item equal to its order quantity.

(4) Initialize the day counter.

(5) Set the item index to first item.

(6) If an order is due from the supplier for this item, go to step 8.

(7) Go to step 11.

(8) Increase the stock level by one order quantity.

(9) Remove the note from the record that an order is outstanding.

(10) Compute and record keeping costs of the receipt from the current day until the next issue.

(11) If this item is due to have an issue made from stock, go to step 13.

(12) Go to step 18.

(13) Determine the size of the present issue.

(14) Decrease the stock level by the amount determined in step 13.

(15) Determine when the next issue is to be.

(16) Go to step 13 if there is to be an issue on the current day.

(17) Note in the item record, the date of the next issue.

(18) Compute and record the keeping costs of the present inventory between now and the next issue.

(19) If the present stock level is at, or below, the order point, or if the family Boolean digit equals one and the stock level is near enough to the order point that additional keeping costs are less than a given constant, go to step 21.

(20) Go to step 27.

(21) Determine the length of the lead time to be experienced by the current order.

(22) Note in the item record that an order is outstanding and its due date.

(23) If another item in the same family is already on order, denoted by the family Boolean digit, go to step 26.

(24) Set the family Boolean digit equal to one.

(25) Note in the item record that this item incurs the fixed costs associated with an order.

(26) Note in the item record that this item incurs the variable costs associated with an order.

(27) If all items of the inventory have not been considered for the current day, go to step 31.

(28) Increase the day count by one.

(29) If the current day is the end of a week, go to step 33.

(30) If the current day is the end of a year, go to step 36.

(31) Increase the item index to the next item.

(32) Go to step 6.

(33) Compute and record the ordering costs for all items.

(34) Set the family Boolean digit equal to zero for each family.

(35) Go to step 30.

(36) Print values of annual keeping and ordering costs for each item.

(37) Print values of annual summary of keeping and ordering costs for each family.

(38) If the current day is the end of the five-year period, go to step 40.

(39) Go to step 5.

(40) Stop.

CHAPTER IV

RESULTS OF THE STUDY

General

Each of the three Monte Carlo simulations produces a set of data on ordering costs and keeping costs. The three models began simulating inventory operating conditions with identical beginning inventories. Thereafter, the random variates took on values as produced by their respective generators. As inventory activity was simulated, costs were accumulated by calculating keeping costs on a daily basis, and ordering costs as they occurred.

In general, the results of the simulations indicate stability in the models. The fact that keeping costs are greater than ordering costs is due to carrying a reasonably high level of protective stocks.

Table 2 summarizes the annual costs experienced in each model. From Table 3, a grand summary, it is obvious that:

(1) Model II, the multiple-item ordering system, shows a reduction in total costs as compared to Model I.

(2) Model III, the optional-multiple-item ordering system, shows a significant reduction in ordering costs as compared to Model II, but only a slight reduction in total costs due to a corresponding increase in keeping costs experienced in Model III.

(3) The reduction in costs of Model II over Model I are attributable almost wholly to Family I, the large, fast moving, low unit cost family.

TABLE 2. ANNUAL COSTS SUMMARY

FAMILY	YEAR	KEEPING COSTS	ORDERING COSTS
		MODEL I	
1	1	909.91	512.00
1	2	848.39	614.00
1	3	842.91	616.00
1	4	786.46	568.00
1	5	853.09	552.00
2	1	152.92	48.00
2	2	194.14	32.00
2	3	97.17	32.00
2	4	135.20	48.00
2	5	96.06	40.00
3	1	136.23	56.00
3	2	113.64	64.00
3	3	96.28	72.00
3	4	120.01	80.00
3	5	107.86	88.00
		MODEL II	
1	1	825.76	390.00
1	2	819.02	408.00
1	3	780.46	432.00
1	4	817.35	387.00
1	5	789.63	414.00
2	1	187.90	51.00
2	2	94.91	40.00
2	3	123.91	38.00
2	4	139.01	48.00
2	5	114.30	43.00
3	1	118.45	65.00
3	2	100.67	48.00
3	3	114.74	92.00
3	4	110.53	72.00
3	5	128.06	86.00
		MODEL III	
1	1	901.97	314.00
1	2	765.35	239.00
1	3	860.09	370.00
1	4	926.90	380.00
1	5	938.81	345.00
2	1	199.03	32.00
2	2	92.50	35.00
2	3	108.85	32.00
2	4	82.12	34.00
2	5	102.05	56.00
3	1	124.53	51.00
3	2	121.35	52.00
3	3	117.23	73.00
3	4	123.97	70.00
3	5	123.75	78.00

TABLE 3. GRAND SUMMARY OF INVENTORY COSTS

FAMILY	MODEL	KEEPING COSTS	ORDERING COSTS	TOTAL COSTS
1	I	4230.76	2872.00	7102.76
1	II	4032.22	2031.00	6063.22
1	III	4383.12	1748.00	6131.12
2	I	675.49	200.00	875.49
2	II	660.03	220.00	880.03
2	III	548.55	179.00	727.55
3	I	574.02	360.00	943.02
3	II	572.45	363.00	935.45
3	III	610.83	334.00	944.83

(4) The total costs of Family III varied less than $11.00 between all three models.

(5) The total costs of Family I, increased from Model II to Model III, whereas total costs of Family II decreased between the same respective models.

Tables 4 through 18 of Appendix E present more detailed cost data by year, item, and cost category for those who may desire them.

Model II Versus Model I

The lower keeping costs of Model II as compared to Model I resulted primarily from the practice of ordering once a week. This condition had the effect of extending lead time by two and a half days, on the average. As can be seen from Table 2, if this reduction were disallowed on the grounds that it decreased the level of protection against stockouts (which it does), there would still be a saving in order costs of $841.00.

Over the five-year periods, Families II and III experienced very slight increases in ordering costs. The writer deems this increase attributable to the stochastic nature of the systems which resulted in a few more orders under Model II.

The most significant reduction occurred in Family I. The writer infers that the large size of the family coupled with a high degree of activity produced a large proportion of multiple-item orders in Model II. The detailed cost data in Appendix E show that Families II and III had an order frequency of either once or twice per year, with only one exception where one item was ordered three times in one particular year. This seems to substantiate the opinion that small, relatively slow moving families, do not present many opportunities for combination of items on orders, even under the weekly ordering scheme.

Model III Versus Model II

Even though both of these models operated under the weekly ordering rule, a notable increase resulted in Family I keeping costs along with an almost equal decrease in ordering costs. As in Model II, multiple-item ordering system, it appears that the lower ordering costs accrued from the family characteristics favorable to combination: large family size and high rate of activity.

Each item which was optionally-multiple-item ordered increased the keeping costs of that item due to the effective increase in inventory levels caused by ordering before the stock level Φ fell to, or below, its order point OP_i:

$$\Phi_i \leq OP_i$$

This same trend is observed in Family III, the next largest and most active family. Family II, however, seems to stand out with a reduction in both keeping costs and ordering costs, thus lowering total costs about 17 per cent in Model III as compared to Model II. This family is the smallest, least active of the three.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

General Conclusions

The hypothesis that multiple-item ordering systems are more economical than are single-item ordering systems was substantiated.

The present study also revealed that, for particular types of inventory families, optional-multiple-item ordering systems are more economical than are single-item or multiple-item ordering systems.

Further, it was shown that the effects of the two systems are complimentary when they are applied to mutually exclusive families.

Multiple-Item Ordering

Multiple-item ordering provides for the combination of items of the same inventory family in a single purchase order, thereby reducing the outlay of fixed costs associated with placing individual purchase orders for each item.

The present study shows that these economical advantages are of a practical nature and easy to obtain. It appears that little is required in the way of systems changes, as regards a conventional single-item ordering system, to convert to the multiple-item ordering system. Only two accommodations appear to be needed for implementing such a system:

(1) Establishment of inventory families.

(2) Adoption of an item-order combining technique in the ordering routine.

Defining Families

An inventory family may be defined to consist of items which:

(1) Are of the same generic kind.

(2) Have the same supplier source.

(3) Have the same lead time.

(4) Have the same cash terms.

(5) Have the same shipping method.

(6) Have the same mode of storage.

(7) Differ from each other only in physical dimensions or specifications.

It is obvious, of course, that a family may be defined with fewer or greater limitations, and those above are suggestive of those likely to be found in an highly automated inventory accounting system.

Item-Order Combination

Items in a family are subject to stock-level inspection by either manual or automatic devices. When the items are being reviewed for order status, there must be devised a method for "flagging" each item of the family which is to be ordered, so that it may be combined with others which must also be ordered--in the event that there are such.

The present study utilized a computer with a memory storage capacity sufficiently large enough to contain the master inventory records (data) of each entire family. This made it possible to inspect the items serially and use a Boolean number switch for incurring the proper order costs. If the item was to be ordered, the Boolean digit was interrogated to determine if some other family member were already on order. If the current item was the one initiating an order, the

fixed costs were incurred by it, and the Boolean digit was set equal to one. At the conclusion of the ordering routine, the Boolean digit was set equal to zero again.

There are many algorithms which will satisfy this need. The design of such algorithms for particular implementations would be largely predicated upon localized conditions. For this reason, the writer is of the opinion that no useful purpose could be served by further exploration of possibilities in this present paper.

Optional-Multiple-Item Ordering

The writer concludes that there is indeed economic justification for ordering an item prior to the time it reaches, or falls below, its order point.

Before the present study was undertaken, the writer was of the opinion that economic efficiencies would almost certainly result from optionally ordered items in the large families, and possibly in the smaller ones to some extent. The simulations of the present study revealed the opposite to be true, which, after looking back, seems quite obvious.

The high rate of usage experienced by the many items of the large family results in frequent ordering activity. This acts to require that an order be written in nearly every order period. Since an order is likely to be required in any case, there is no point in moving up the ordering event of another item which could just as safely be ordered during a subsequent period on a multiple-item order. The results verify that the tendency to increase keeping costs exceeds the inclination for increased ordering efficiency.

The small, inactive families do not stochastically generate high probabilities for random combination of items on orders. Because of this environment, some additional keeping costs are justified by the elimination of a subsequent single-item order.

Mutually Exclusive Assignment

The present study indicates that multiple-item ordering systems are more economically efficient than optional-multiple-item ordering systems for large, active inventory families, and vice versa for small, inactive families. Therefore, it appears that the most economically efficient management system would be the mutually exclusive assignment to their respective efficiencies. Those families subject to one ordering system would be excluded from the other.

Recommendations

The objective of this present study has been realized with some degree of satisfaction. The method employed was indeed useful in stimulating insight into and objective evaluation of a situation which does not lend itself to closed form solution.

Additional studies could be directed toward determination of decision rules for assigning families to the two ordering systems. The techniques of Monte Carlo simulation appear to be applicable to that determination.

There are several inventory policies which a firm may follow, but little is known about their relative merits. A comparative study of the effects of such policies would be a potential significant contribution to inventory theory and operation.

Another interesting study could be an experimental investigation of the length of time between writing orders. The present study, for instance, arbitrarily established that orders would be placed once each week. A study may show that a longer or shorter period, with given systems costs, may be more economical.

A comprehensive study could be conducted in which the models are replicated through a series of designed experiments, perhaps factorial, to determine the optimal decision rules relative to each parameter affecting inventory costs.

The present writer has in the past devoted a great deal of thought to the propositions pursued in this study without being able to reach more than an intuitive conclusion. It is felt that the subject has been clarified to an extent sufficient to warrant serious consideration of the practical application of both the multiple-item and optional-multiple-item ordering systems to the management of inventory. It is also hoped that the application and implementation of the Monte Carlo simulation methods to the present study will prove useful to others in exploring problems of a similar nature.

APPENDIX A

SAMPLE INVENTORY FILE PROGRAM

```
        FLOATING SUM$
        FLOATING AILT, RADICAL $
        FLOATING V$
        INTEGER OTHERWISE$
        ARRAY A(25), B(25), DESCRIPTION(3)$
        INPUT DATA(IDNUMBER,
        FOR I = (1,1,3)$ DESCRIPTION (I),
        EXPECTEDANNUALUSAGE, UNITCOST,
        EXPECTEDNUMBERISSUES, LEADTIME)$
        MAGREWIND(3,0)$
        WRITE($$H2)$
        WRITE($$H3)$
        C = 0$
START.. READ($$DATA)$
        IF IDNUMBER EQL 999$
BEGIN   FOR I = (1,1,25)$ A(I) = 999$
        OVERWRITE(3,25,0$A(1))$
        MAGBACK(3,1)$
        MAGREAD(3,25$B(1))$
        MAGREWIND(3,0)$
        STOP 9999
        END$  AILT = (FLOAT(LEADTIME)/ 364.0) .
        FLOAT(EXPECTEDNUMBERISSUES)$
        SUM = 0.0$
        K =0.0$
        V = 2.71828*-AILT$
AGAIN.. SUM = SUM + V$
        IF SUM GEQ 0.95$
        GO AHEAD$
        K = K + 1.0$
        V = V.(AILT / K)$
        GO AGAIN$
AHEAD.. IF K EQL 0$
BEGIN   OP = 0$
        GO RAD          END$
        OP = K . (EXPECTEDANNUALUSAGE / EXPECTEDNUMBERISSUES)$
RAD..   RADICAL = (((2). EXPECTEDANNUALUSAGE).(8.00)) /
        ((UNITCOST / 10000.0) . (0.2))$
        EOQ = SQRT(RADICAL)$
        A(1) = IDNUMBER$
```

```
          A(2) = DESCRIPTION(1)$
          A(3) = DESCRIPTION(2)$
          A(4) = DESCRIPTION(3)$
          A(5) = EXPECTEDANNUALUSAGE$
          A(6) = UNITCOST$
          A(7) = EXPECTEDNUMBERISSUES$
          A(8) = LEADTIME$
          A(9) = EOQ$
          A(10) = OP$
          OVERWRITE(3,25,0$A(1))$
          MAGBACK(3,1)$
          MAGREAD(3,25$B(1))$
          WRITE($$W1,H1)$
          C = C + 1 $
          IF C EQL 20$
    BEGIN WRITE($$H2)$  WRITE($$H3)$ C = 0$ END$
          GO TO START$
FORMAT H1(I5, B2,A15, B1,I5,S8.4,B2,I4,B2,I4,B1,
          I5,B1,I5,W4),
          H2(*THE INITIAL INVENTORY AND ITEM PARAMETERS*,W3),
          H3(*ID NO.   DESCRIPTION   USAGE  COST   ISSUES  *
*LT   EOQ   OP*, W4)$
OUTPUT W1(FOR I = (1,1,5)$ A(I), (FLOAT(A(6)) / 10000.0),
          FOR I = (7,1,10)$ A(I))$
FINISH$
```

APPENDIX B

MONTE CARLO MODEL I PROGRAM

```
        FLOATING PR,RT,OT  $
        FLOATING FIXED, VAR,TKC(),TOC(),SUM,V,TKCI1, TOCI1,
        TKCI2,TOCI2,TKCI3,TOCI3$
        INTEGER OTHERWISE $
        ARRAY R(1250), TKC(50), TOC(50)$
        MAGREWIND(3,0)$
        RHO = 5*13$
        RB  = 3175492873 $
        FOR I = (0,1,46)$
BEGIN   MAGREAD(3,25$ R(I.(25)+1))$
        R(I.(25)+13) = R(I.(25)+9)$
 END$
        DAY = 0$
        YEAR = 260$
        M =0$   N= 45$
START..FOR I = (M,1,N)$
BEGIN   IF (R(I.(25)+12) GTR 0) AND (R(I.(25)+12) LEQ DAY)$
BEGIN   R(I.(25)+13) = R(I.(25)+13) + R(I.(25)+9)$
   TKC(I+1)=TKC(I+1) +((R(I.(25)+ 9).(R(I.(25)+6).(0.0001))).
        (0.00076923076))
        .(R(I.(25)+15)- DAY) $
        R(I.(25)+12)=0$ R(I.(25)+11)=0 $ END$
ISSUE..IF R(I.(25)+15) EQL DAY $
        GO SIZE  $
        GO DUMMY $
GENIS..RB = RB . RHO $
        PR = (1.0**-10) . FLOAT(RB)$
        RT=(-(LOG(PR))/(FLOAT(R(I.(25)+7))/260.0))$
        IF RT GTR 1.0 $
        GO COST  $
        GO SIZE  $
STACT..R(I.(25)+ 13) = R(I.(25)+13) - J $
        R(I.(25)+16)=R(I.(25)+16)+1$
        R(I.(25)+17)=R(I.(25)+17)+J$
        GO GENIS $
COST.. R(I.(25)+ 15) = RT + DAY $
   TKC(I+1)=TKC(I+1)  + ((R(I.(25)+13). (R(I.(25)+ 6).
        (0.0001))). (0.00076923076)).RT$
        IF (R(I.(25)+13) LSS R(I.(25)+10)) AND (R(I.25)+11)
        EQL 0)$      GO GENOD$
```

```
        GO DUMMY $
 SIZE..EIS = R(I.(25)+5) / R(I.(25)+7)$
GENSI..RB=RB . RHO$
        PR = (1.0**-10) . FLOAT(RB)$
        V = -EIS(LOG(PR))$
        J=V + 0.5$
 STOUT..IF J GTR R(I.(25)+13)$
 BEGIN  R(I.(25)+14) = R(I.(25)+14) + 1 $
        GO GENIS$
 END$   GO STACT $
 GENOD..RB = RB . RHO$
        PR = (1.0**-10) . FLOAT(RB)$
        OT=(-(LOG(PR))/(1.0/FLOAT(R(I.(25)+8))))$
        IF OT GTR 2.0 $
 BEGIN  R(I.(25)+11)= DAY $
        R(I.(25) + 18)  = R(I.(25)+18) + 1$
        R(I.(25)+12) = DAY + OT $  GO ODCO $
 END$   GO GENOD $
 DUMMY..END $
        DAY = DAY + 1 $
 DODAY..IF  DAY GTR 1300 $
        GO EOJ $
        IF DAY  GEQ YEAR $
 BEGIN  YEAR = DAY + 260 $
        GO ANNUAL$ END $
        GO START $
 ODCO.. TOC(I+1)=TOC(I+1) + 8.0$
        GO DUMMY $
 ANNUAL..WRITE ($$W1,H1)$
        FOR L = (1,1, 31)$
 BEGIN   TOCI1 = TOCI1 + TOC(L) $
         TKCI1 = TKCI1 + TKC(L) $
 END $
         FOR L = (32,1,36)$
 BEGIN   TOCI2 = TOCI2 + TOC(L)$
         TKCI2 = TKCI2 + TKC(L)$
 END $
        FOR L = (37,1,46)$
 BEGIN   TOCI3 = TOCI3 + TOC(L)$
         TKCI3 = TKCI3 + TKC(L)$
 END $
         WRITE ($$W2,H2)$
         FOR L = (1,1,46)$ BEGIN TOC(L)=0$ TKC(L)=0$ END$
        GO START $
   EOJ.. STOP 9999$
FORMAT H1(*THE INDIVIDUAL KEEPING COST ON DAY*,I6,* ARE AS *,
        *FOLLOWS*, W3,
        4(10X12.2 ,W0), 6X12.2, W0 ,
          *THE INDIVIDUAL ORDERING COST ON THE SAME PERIOD *,
        *ARE*, W4,
```

```
        4(10X12.2 ,W0), 6X12.2, W0) ,
      H2(*THE TOTAL KEEPING COST FOR FAMILY 1 IS *,X12.2, W3,
         *THE TOTAL ORDERING COST FOR FAMILY 1 IS *,X11.2,W4,
         *THE TOTAL KEEPING COST FOR FAMILY 2 IS *,X12.2, W4,
         *THE TOTAL ORDERING COST FOR FAMILY 2 IS *,X11.2, W4,
         *THE TOTAL KEEPING COST FOR FAMILY 3 IS *, X12.2,W4,
         *THE TOTAL ORDERING COST FOR FAMILY 3 IS *, X11.2,W4)$
OUTPUT W1(DAY, FOR L =(1,1,46)$ TKC(L), FOR L=(1,1,46)$ TOC(L)),
       W2(TKCI1,TOCI1,TKCI2,TOCI2,TKCI3,TOCI3)$
       FINISH $
```

APPENDIX C

MONTE CARLO MOFEL II PROGRAM

```
        FLOATING PR,RT,OT  $
        FLOATING FIXED, VAR,TKC(),TOC(),SUM,V,TKCI1, TOCI1,
        TKCI2,TOCI2,TKCI3,TOCI3$
        INTEGER OTHERWISE $
        ARRAY R(1250), TKC(50), TOC(50)$
        MAGREWIND(3,0)$
        RHO = 5*13$
        RB  = 3175492873 $
        FOR I = (0,1,46)$
BEGIN   MAGREAD(3,25$ R(I.(25)+1))$
        R(I.(25)+13) = R(I.(25)+9)$
 END$
        DAY = 0$
        WEEK = 5$
        YEAR = 260$
        SW1=0$  SW2=0$  SW3=0$
        M =0$   N= 45$
START..FOR I = (M,1,N)$
BEGIN   IF (R(I.(25)+12) GTR 0) AND (R(I.(25)+12) LEQ DAY)$
BEGIN   R(I.(25)+13) = R(I.(25)+13) + R(I.(25)+9)$
   TKC(I+1)=TKC(I+1) +((R(I.(25)+ 9).(R(I.(25)+6).(0.0001))).
        (0.000076923076))
        .(R(I.(25)+15)- DAY) $
        R(I.(25)+12)=0$ R(I.(25)+11)=0 $ END$
ISSUE..IF R(I.(25)+15) EQL DAY $
        GO SIZE  $
        GO DUMMY $
GENIS..RB = RB . RHO $
        PR = (1.0**-10) . FLOAT(RB)$
        RT=(-(LOG(PR))/(FLOAT(R(I.(25)+7))/260.0))$
        IF RT GTR 1.0 $
        GO COST  $
        GO SIZE  $
STACT..R(I.(25)+ 13) = R(I.(25)+13) - J $
        R(I.(25)+16)=R(I.(25)+16)+1$
        R(I.(25)+17)=R(I.(25)+17)+J$
        GO GENIS $
COST.. R(I.(25)+ 15) = RT + DAY $
   TKC(I+1)=TKC(I+1)  + ((R(I.(25)+13). (R(I.(25)+ 6).
        (0.0001))). (0.000076923076)).RT$
```

```
        IF (R(I.(25)+13) LSS R(I.(25)+10)) AND (R(I.25)+11)
        EQL 0)$      GO GENOD$
GENSI..RB=RB . RHO$
        PR = (1.0**-10) . FLOAT(RB)$
        V = -EIS(LOG(PR))$
        J=V + 0.5$
 STOUT..IF J GTR R(I.(25)+13)$
 BEGIN  R(I.(25)+14) = R(I.(25)+14) + 1 $
        GO GENIS$
 END$   GO STACT $
 GENOD..RB = RB . RHO$
        PR = (1.0**-10) . FLOAT(RB)$
        OT=(-(LOG(PR))/(1.0/FLOAT(R(I.(25)+8))))$
        IF OT GTR 2.0 $
BEGIN R(I.(25)+11) = DAY + (WEEK - DAY)$
        R(I.(25)+12)= DAY + OT + (WEEK - DAY)$
        IF I LEQ 31$
  BEGIN  IF SW1 EQL 1$
  BEGIN  R(I.(25)+20)= R(I.(25)+20) + 1$
        GO DUMMY$ END$
         R(I.(25)+20)= R(I.(25)+20) + 1$
        R(I.(25)+19) = R(I.(25)+19)+1$
        SW1 = 1$
        GO DUMMY$ END$
        IF I LEQ 36$
  BEGIN  IF SW2 EQL 1$
  BEGIN  R(I.(25)+20)= R(I.(25)+20) + 1$
        GO DUMMY$ END$
         R(I.(25)+20)= R(I.(25)+20) + 1$
        R(I.(25)+19) = R(I.(25)+19)+1$
         SW2 = 1$
        GO DUMMY$ END$
        IF SW3 EQL 1$
  BEGIN  R(I.(25)+20)= R(I.(25)+20) + 1$
        GO DUMMY$ END$
         R(I.(25)+20)= R(I.(25)+20) + 1$
        R(I.(25)+19) = R(I.(25)+19)+1$
         SW3 = 1$
        GO DUMMY$
 END$   GO GENOD $
 DUMMY..END $
        DAY = DAY + 1 $
        IF DAY GEQ WEEK$
  BEGIN  WEEK = WEEK + 5$  GO ODCO$ END$
 DODAY..IF  DAY GTR 1300 $
        GO EOJ $
        IF DAY  GEQ YEAR $
 BEGIN  YEAR = DAY + 260 $
        GO ANNUAL$ END $
        GO START $
```

```
        FOR L = (0,1,45)$
  BEGIN  R(L.(25)+19) =0$  R(L.(25)+20)=0$ END$
        SW1=0$  SW2=0$  SW3=0$
        GO DODAY$
 ANNUAL..WRITE ($$W1,H1)$
        FOR L = (1,1, 31)$
 BEGIN   TOCI1 = TOCI1 + TOC(L) $
         TKCI1 = TKCI1 + TKC(L) $
 END $
         FOR L = (32,1,36)$
 BEGIN   TOCI2 = TOCI2 + TOC(L)$
         TKCI2 = TKCI2 + TKC(L)$
 END $
        FOR L = (37,1,46)$
 BEGIN   TOCI3 = TOCI3 + TOC(L)$
         TKCI3 = TKCI3 + TKC(L)$
 END $
         WRITE ($$W2,H2)$
        GO START $
         FOR L = (1,1,46)$ BEGIN TOC(L)=0$ TKC(L)=0$ END$
   EOJ.. STOP 9999$
FORMAT H1(*THE INDIVIDUAL KEEPING COST ON DAY*,I6,* ARE AS *,
        *FOLLOWS*, W3,
        4(10X12.2 ,W0), 6X12.2, W0 ,
          *THE INDIVIDUAL ORDERING COST ON THE SAME PERIOD *,
        *ARE*, W4,
        4(10X12.2 ,W0), 6X12.2, W0) ,
      H2(*THE TOTAL KEEPING COST FOR FAMILY 1 IS *,X12.2, W3,
         *THE TOTAL ORDERING COST FOR FAMILY 1 IS *,X11.2,W4,
         *THE TOTAL KEEPING COST FOR FAMILY 2 IS *,X12.2, W4,
         *THE TOTAL ORDERING COST FOR FAMILY 2 IS *,X11.2, W4,
         *THE TOTAL KEEPING COST FOR FAMILY 3 IS *, X12.2,W4,
         *THE TOTAL ORDERING COST FOR FAMILY 3 IS *, X11.2,W4)$
OUTPUT W1(DAY, FOR L =(1,1,46)$ TKC(L), FOR L=(1,1,46)$ TOC(L)),
       W2(TKCI1,TOCI1,TKCI2,TOCI2,TKCI3,TOCI3)$
        FINISH $
```

APPENDIX D

MONTE CARLO MODEL III PROGRAM

```
        FLOATING PR,RT,OT  $
        FLOATING FIXED, VAR,TKC(),TOC(),SUM,V,TKCI1, TOCI1,
        TKCI2,TOCI2,TKCI3,TOCI3$
        INTEGER OTHERWISE $
        ARRAY R(1250), TKC(50), TOC(50)$
        MAGREWIND(3,0)$
        RHO = 5*13$
        RB  = 3175492873 $
        FOR I = (0,1,46)$
BEGIN   MAGREAD(3,25$ R(I.(25)+1))$
        R(I.(25)+13) = R(I.(25)+9)$
 END$
        DAY = 0$
        WEEK = 5$
        YEAR = 260$
        SW1=0$  SW2=0$  SW3=0$
        M =0$   N= 45$
START..FOR I = (M,1,N)$
BEGIN   IF (R(I.(25)+12) GTR 0) AND (R(I.(25)+12) LEQ DAY)$
BEGIN   R(I.(25)+13) = R(I.(25)+13) + R(I.(25)+9)$
   TKC(I+1)=TKC(I+1) +((R(I.(25)+ 9).(R(I.(25)+6).(0.0001))).
        (0.00076923076))
        .(R(I.(25)+15)= DAY) $
        R(I.(25)+12)=0$ R(I.(25)+11)=0 $ END$
ISSUE..IF R(I.(25)+15) EQL DAY $
        GO SIZE  $
        GO DUMMY $
GENIS..RB = RB . RHO $
        PR = (1.0**-10) . FLOAT(RB)$
        RT=(-(LOG(PR))/(FLOAT(R(I.(25)+7))/260.0))$
        IF RT GTR 1.0 $
        GO COST  $
        GO SIZE  $
STACT..R(I.(25)+ 13) = R(I.(25)+13) - J $
        R(I.(25)+16)=R(I.(25)+16)+1$
        R(I.(25)+17)=R(I.(25)+17)+J$
        GO GENIS $
COST.. R(I.(25)+ 15) = RT + DAY $
   TKC(I+1)=TKC(I+1)  + ((R(I.(25)+13). (R(I.(25)+ 6).
        (0.0001))). (0.00076923076)).RT$
```

```
        IF (R(I.(25)+13) LSS R(I.(25)+10)) AND (R(I.25)+11)
        EQL 0)$      GO GENOD$
        DELTA = ((6.0)/(((R(I.(25)+6).(0.0001)).(0.2)))) +
                R(I.(25)+10)$
        IF  (I LSS 31) AND (SW1 EQL 1)$  GO CANCK$
        IF (I GTR 30) AND (I LSS 36) AND (SW2 EQL 1)$  GO CANCK$
        IF (I GTR 35) AND (SW3 EQL 1)$  GO CANCK$
        GO DUMMY$
CANCK.. IF (R(I.(25)+13) LSS DELTA) AND (R(I.(25)+11) EQL 0) $
        GO GENOD $
        GO DUMMY $
 SIZE..EIS = R(I.(25)+5) / R(I.(25)+7)$
GENSI..RB=RB . RHO$
        PR = (1.0**-10) . FLOAT(RB)$
        V = -EIS(LOG(PR))$
        J=V + 0.5$
 STOUT..IF J GTR R(I.(25)+13)$
 BEGIN  R(I.(25)+14) = R(I.(25)+14) + 1 $
        GO GENIS$
 END$   GO STACT $
 GENOD..RB = RB . RHO$
        PR = (1.0**-10) . FLOAT(RB)$
        OT=(-(LOG(PR))/(1.0/FLOAT(R(I.(25)+8))))$
        IF OT GTR 2.0 $
BEGIN R(I.(25)+11) = DAY + (WEEK - DAY)$
        R(I.(25)+12)= DAY + OT + (WEEK - DAY)$
        IF I LEQ 31$
  BEGIN  IF SW1 EQL 1$
  BEGIN  R(I.(25)+20)= R(I.(25)+20) + 1$
        GO DUMMY$ END$
         R(I.(25)+20)= R(I.(25)+20) + 1$
        R(I.(25)+19) = R(I.(25)+19)+1$
        SW1 = 1$
        GO DUMMY$ END$
        IF I LEQ 36$
  BEGIN  IF SW2 EQL 1$
  BEGIN  R(I.(25)+20)= R(I.(25)+20) + 1$
        GO DUMMY$ END$
         R(I.(25)+20)= R(I.(25)+20) + 1$
        R(I.(25)+19) = R(I.(25)+19)+1$
         SW2 = 1$
        GO DUMMY$ END$
        IF SW3 EQL 1$
  BEGIN  R(I.(25)+20)= R(I.(25)+20) + 1$
        GO DUMMY$ END$
         R(I.(25)+20)= R(I.(25)+20) + 1$
        R(I.(25)+19) = R(I.(25)+19)+1$
         SW3 = 1$
        GO DUMMY$
 END$   GO GENOD $
```

```
DUMMY..END $
        DAY = DAY + 1 $
        IF DAY GEQ WEEK$
 BEGIN  WEEK = WEEK + 5$  GO ODCO$ END$
DODAY..IF  DAY GTR 1300 $
        GO EOJ $
        IF DAY  GEQ YEAR $
BEGIN   YEAR = DAY + 260 $
        GO ANNUAL$ END $
        GO START $
ODCO..  IF SW1 EQL 1 $
BEGIN   FOR L=(0,1,30)$
 TOC(L+1)=TOC(L+1)+(5.0)(R(L.(25)+19))+(3.0)(R(L.(25)+20))$
        END $
        IF SW2 EQL 1 $
BEGIN   FOR L=(31,1,35)$
 TOC(L+1)=TOC(L+1)+(5.0)(R(L.(25)+19))+(3.0)(R(L.(25)+20))$
        END $
        IF SW3 EQL 1 $
BEGIN   FOR L=(36,1,45)$
 TOC(L+1)=TOC(L+1)+(5.0)(R(L.(25)+19))+(3.0)(R(L.(25)+20))$
        END $
        FOR L = (0,1,45)$
 BEGIN  R(L.(25)+19) =0$  R(L.(25)+20)=0$ END$
        SW1=0$  SW2=0$  SW3=0$
        GO DODAY$
ANNUAL..WRITE ($$W1,H1)$
        FOR L = (1,1, 31)$
BEGIN    TOCI1 = TOCI1 + TOC(L) $
         TKCI1 = TKCI1 + TKC(L) $
END $
         FOR L = (32,1,36)$
BEGIN    TOCI2 = TOCI2 + TOC(L)$
         TKCI2 = TKCI2 + TKC(L)$
END $
        FOR L = (37,1,46)$
BEGIN    TOCI3 = TOCI3 + TOC(L)$
         TKCI3 = TKCI3 + TKC(L)$
END $
         WRITE ($$W2,H2)$
         FOR L = (1,1,46)$ BEGIN TOC(L)=0$ TKC(L)=0$ END$
        GO START $
  EOJ.. STOP 9999$
FORMAT H1(*THE INDIVIDUAL KEEPING COST ON DAY*,I6,* ARE AS *,
        *FOLLOWS*, W3,
        4(10X12.2 ,W0), 6X12.2, W0 ,
          *THE INDIVIDUAL ORDERING COST ON THE SAME PERIOD *,
        *ARE*, W4,
        4(10X12.2 ,W0), 6X12.2, W0) ,
      H2(*THE TOTAL KEEPING COST FOR FAMILY 1 IS *,X12.2, W3,
```

```
          *THE TOTAL ORDERING COST FOR FAMILY 1 IS *,X11.2,W4,
          *THE TOTAL KEEPING COST FOR FAMILY 2 IS *,X12.2, W4,
          *THE TOTAL ORDERING COST FOR FAMILY 2 IS *,X11.2, W4,
          *THE TOTAL KEEPING COST FOR FAMILY 3 IS *, X12.2,W4,
          *THE TOTAL ORDERING COST FOR FAMILY 3 IS *, X11.2,W4)$
OUTPUT W1(DAY, FOR L =(1,1,46)$ TKC(L), FOR L=(1,1,46)$ TOC(L)),
       W2(TKCI1,TOCI1,TKCI2,TOCI2,TKCI3,TOCI3)$
       FINISH $
```

APPENDIX E

DETAILED COST DATA

TABLE 4. MODEL I, YEAR 1

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	26.65	16.00
102	14.92	8.00
103	24.49	16.00
104	28.07	16.00
105	13.20	8.00
106	19.72	16.00
107	32.93	32.00
108	18.04	8.00
109	21.68	16.00
110	16.28	16.00
111	19.07	16.00
112	25.64	16.00
113	31.95	24.00
114	33.04	16.00
115	18.77	8.00
116	10.41	0.00
117	13.83	8.00
118	11.15	8.00
119	55.89	16.00
120	23.81	16.00
121	73.32	56.00
122	118.35	32.00
123	15.85	8.00
124	7.62	0.00
125	24.88	16.00
126	18.12	16.00
127	31.19	16.00
128	46.31	32.00
129	68.33	32.00
130	38.35	24.00
131	7.89	0.00
201	23.15	8.00
202	14.31	0.00
203	18.30	8.00
204	43.31	16.00
205	53.82	16.00
301	9.64	8.00
302	8.67	0.00
303	18.30	8.00
304	16.28	8.00
305	12.19	0.00
306	19.80	8.00
307	15.72	8.00
308	15.13	16.00
309	12.18	0.00
310	8.27	0.00

TABLE 5. MODEL I, YEAR 2

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	27.76	24.00
102	11.17	16.00
103	19.23	16.00
104	19.01	24.00
105	15.11	16.00
106	17.56	16.00
107	31.94	16.00
108	24.14	8.00
109	23.56	16.00
110	18.14	8.00
111	18.23	16.00
112	27.16	8.00
113	32.54	24.00
114	34.65	24.00
115	3.67	0.00
116	5.68	8.00
117	14.22	16.00
118	10.97	8.00
119	62.30	24.00
120	18.29	24.00
121	84.81	56.00
122	101.78	56.00
123	16.87	8.00
124	3.90	8.00
125	24.48	24.00
126	17.96	8.00
127	22.55	16.00
128	44.87	32.00
129	56.22	56.00
130	29.01	32.00
131	10.46	16.00
201	17.81	16.00
202	14.49	0.00
203	38.42	0.00
204	63.85	8.00
205	59.55	8.00
301	5.87	8.00
302	7.95	8.00
303	11.38	8.00
304	17.38	8.00
305	4.17	0.00
306	14.30	0.00
307	8.55	0.00
308	13.40	16.00
309	11.88	8.00
310	18.72	8.00

TABLE 6. MODEL I, YEAR 3

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	23.88	24.00
102	12.32	8.00
103	24.92	24.00
104	14.57	24.00
105	13.96	8.00
106	17.49	16.00
107	32.94	24.00
108	17.51	8.00
109	20.75	24.00
110	17.95	8.00
111	17.61	8.00
112	25.15	24.00
113	28.96	32.00
114	24.43	24.00
115	19.78	16.00
116	7.82	0.00
117	17.43	16.00
118	11.83	8.00
119	55.22	48.00
120	22.03	16.00
121	70.29	48.00
122	100.56	48.00
123	14.42	8.00
124	4.52	0.00
125	21.48	24.00
126	20.33	16.00
127	31.61	8.00
128	53.90	32.00
129	50.05	40.00
130	36.00	24.00
131	13.04	8.00
201	25.33	16.00
202	4.64	8.00
203	14.31	0.00
204	0.00	0.00
205	52.87	8.00
301	6.58	0.00
302	4.10	0.00
303	6.98	8.00
304	16.54	16.00
305	7.55	8.00
306	9.97	8.00
307	11.04	8.00
308	16.95	8.00
309	7.87	8.00
310	8.65	8.00

TABLE 7. MODEL I, YEAR 4

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	25.26	24.00
102	17.15	8.00
103	24.64	16.00
104	19.40	16.00
105	11.95	16.00
106	15.39	16.00
107	28.58	16.00
108	14.01	16.00
109	25.50	24.00
110	21.35	8.00
111	15.11	16.00
112	22.97	16.00
113	28.95	24.00
114	30.06	16.00
115	7.41	0.00
116	3.99	8.00
117	23.00	8.00
118	8.93	16.00
119	34.10	16.00
120	16.16	16.00
121	75.04	40.00
122	92.36	32.00
123	18.39	8.00
124	1.22	8.00
125	16.67	24.00
126	15.63	8.00
127	23.39	24.00
128	53.87	32.00
129	50.30	48.00
130	30.99	40.00
131	14.56	8.00
201	43.66	8.00
202	7.78	8.00
203	14.23	0.00
204	17.83	16.00
205	51.67	16.00
301	14.56	8.00
302	11.57	16.00
303	14.17	8.00
304	13.34	8.00
305	6.61	8.00
306	14.49	8.00
307	11.49	8.00
308	18.99	8.00
309	11.44	8.00
310	3.29	0.00

TABLE 8. MODEL I, YEAR 5

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	24.96	24.00
102	13.88	8.00
103	23.14	24.00
104	25.44	8.00
105	14.96	8.00
106	19.61	16.00
107	25.40	24.00
108	15.06	8.00
109	20.45	8.00
110	19.58	8.00
111	16.14	16.00
112	24.26	24.00
113	30.46	32.00
114	28.11	24.00
115	3.49	8.00
116	7.46	8.00
117	14.32	0.00
118	13.19	8.00
119	79.71	8.00
120	18.95	16.00
121	78.33	48.00
122	69.41	56.00
123	19.08	8.00
124	6.78	0.00
125	23.56	24.00
126	18.40	24.00
127	24.89	16.00
128	62.12	32.00
129	59.60	40.00
130	35.25	16.00
131	6.96	8.00
201	7.27	8.00
202	7.46	16.00
203	10.64	0.00
204	19.51	8.00
205	51.14	8.00
301	6.78	8.00
302	2.63	0.00
303	14.07	16.00
304	15.32	16.00
305	3.44	8.00
306	8.37	0.00
307	15.68	16.00
308	14.38	8.00
309	14.65	8.00
310	12.51	8.00

TABLE 9. MODEL II, YEAR 1

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	26.14	24.00
102	12.01	3.00
103	23.24	11.00
104	20.67	11.00
105	13.60	3.00
106	17.84	11.00
107	30.93	22.00
108	15.12	3.00
109	21.45	8.00
110	14.54	11.00
111	17.33	8.00
112	25.44	16.00
113	27.96	19.00
114	25.94	11.00
115	13.37	8.00
116	9.68	0.00
117	15.33	8.00
118	11.51	8.00
119	49.46	20.00
120	16.90	6.00
121	66.80	41.00
122	119.80	33.00
123	11.15	6.00
124	4.76	0.00
125	22.82	11.00
126	20.67	6.00
127	25.91	11.00
128	40.59	25.00
129	63.73	27.00
130	32.42	19.00
131	8.50	0.00
201	41.46	3.00
202	14.12	16.00
203	37.70	8.00
204	42.48	16.00
205	51.80	8.00
301	9.45	8.00
302	6.90	8.00
303	10.25	0.00
304	16.39	16.00
305	10.42	8.00
306	12.97	3.00
307	16.12	16.00
308	12.45	3.00
309	10.89	0.00
310	12.58	3.00

TABLE 10. MODEL II. YEAR 2

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	22.96	11.00
102	13.74	11.00
103	24.83	19.00
104	23.56	16.00
105	11.37	8.00
106	19.96	11.00
107	31.26	14.00
108	13.23	8.00
109	25.01	11.00
110	21.39	11.00
111	15.08	16.00
112	21.71	6.00
113	24.86	17.00
114	24.03	11.00
115	10.17	16.00
116	4.37	0.00
117	16.40	0.00
118	7.60	16.00
119	44.65	14.00
120	18.46	16.00
121	72.47	33.00
122	142.23	36.00
123	16.78	0.00
124	3.65	0.00
125	15.07	16.00
126	17.60	11.00
127	26.69	16.00
128	54.31	25.00
129	44.36	25.00
130	27.69	14.00
131	3.35	0.00
201	22.51	0.00
202	7.10	8.00
203	10.71	0.00
204	16.64	8.00
205	37.91	24.00
301	9.14	8.00
302	4.57	0.00
303	9.31	8.00
304	16.64	0.00
305	11.53	0.00
306	14.10	8.00
307	4.44	0.00
308	13.49	8.00
309	5.95	0.00
310	11.45	16.00

TABLE 11. MODEL II, YEAR 3

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	21.96	22.00
102	11.84	3.00
103	20.78	6.00
104	20.71	19.00
105	14.73	11.00
106	18.92	11.00
107	32.51	24.00
108	20.49	6.00
109	21.20	3.00
110	12.71	11.00
111	18.34	11.00
112	19.25	14.00
113	28.07	24.00
114	23.90	22.00
115	11.98	0.00
116	8.06	8.00
117	15.37	3.00
118	11.48	3.00
119	52.90	6.00
120	16.41	8.00
121	45.51	22.00
122	82.45	38.00
123	18.69	11.00
124	2.17	0.00
125	19.80	6.00
126	21.85	16.00
127	24.52	19.00
128	61.94	17.00
129	52.90	56.00
130	36.79	24.00
131	12.10	8.00
201	10.77	6.00
202	14.69	8.00
203	10.14	8.00
204	34.04	8.00
205	54.26	8.00
301	7.27	8.00
302	10.11	3.00
303	10.59	6.00
304	16.36	16.00
305	7.77	0.00
306	12.40	16.00
307	11.77	16.00
308	16.83	8.00
309	9.73	8.00
310	11.86	11.00

TABLE 12. MODEL II, YEAR 4

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	23.31	8.00
102	14.28	8.00
103	21.79	19.00
104	19.94	11.00
105	16.30	16.00
106	19.96	11.00
107	23.93	11.00
108	18.70	8.00
109	13.40	3.00
110	18.17	24.00
111	17.68	11.00
112	22.08	6.00
113	34.55	19.00
114	24.73	11.00
115	8.43	8.00
116	3.44	3.00
117	7.39	3.00
118	13.52	8.00
119	56.42	11.00
120	17.96	6.00
121	75.93	38.00
122	78.94	38.00
123	24.82	3.00
124	10.72	3.00
125	19.88	22.00
126	15.51	8.00
127	31.90	3.00
128	60.31	20.00
129	61.34	33.00
130	42.14	14.00
131	2.72	0.00
201	38.98	0.00
202	9.13	16.00
203	27.81	8.00
204	23.40	16.00
205	39.66	8.00
301	13.86	8.00
302	4.47	0.00
303	14.26	8.00
304	16.88	8.00
305	2.92	8.00
306	7.75	0.00
307	16.43	8.00
308	18.07	16.00
309	6.15	16.00
310	9.69	0.00

TABLE 13. MODEL II, YEAR 5

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	30.00	22.00
102	11.18	8.00
103	24.06	24.00
104	23.64	8.00
105	15.71	8.00
106	18.69	6.00
107	34.92	22.00
108	14.52	8.00
109	25.76	8.00
110	20.37	8.00
111	13.51	6.00
112	20.52	11.00
113	28.91	14.00
114	29.72	11.00
115	16.09	8.00
116	8.03	8.00
117	18.89	3.00
118	10.10	11.00
119	41.57	25.00
120	15.44	14.00
121	53.05	27.00
122	69.45	30.00
123	8.80	3.00
124	0.00	0.00
125	22.62	16.00
126	20.31	11.00
127	33.42	9.00
128	63.49	17.00
129	63.59	38.00
130	31.08	22.00
131	2.05	8.00
201	12.33	8.00
202	21.12	0.00
203	19.26	16.00
204	21.89	8.00
205	39.67	11.00
301	7.99	8.00
302	11.59	3.00
303	9.51	8.00
304	13.54	8.00
305	11.28	8.00
306	16.67	16.00
307	14.21	8.00
308	12.97	8.00
309	15.18	3.00
310	15.06	16.00

TABLE 14. MODEL III, YEAR 1

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	26.22	19.00
102	15.96	0.00
103	17.76	6.00
104	28.63	6.00
105	15.69	3.00
106	21.80	14.00
107	32.24	6.00
108	19.09	3.00
109	29.89	16.00
110	19.65	16.00
111	19.78	3.00
112	28.58	11.00
113	35.35	6.00
114	33.42	9.00
115	18.70	3.00
116	11.59	3.00
117	15.56	16.00
118	14.22	6.00
119	49.82	14.00
120	15.72	6.00
121	62.89	25.00
122	103.41	23.00
123	24.21	8.00
124	10.26	8.00
125	21.73	19.00
126	22.20	3.00
127	25.15	6.00
128	64.77	30.00
129	52.40	12.00
130	33.45	14.00
131	11.68	0.00
201	37.51	0.00
202	14.97	0.00
203	32.03	0.00
204	70.04	24.00
205	44.47	8.00
301	5.87	0.00
302	13.51	0.00
303	11.11	3.00
304	13.85	16.00
305	9.74	8.00
306	16.65	0.00
307	12.62	8.00
308	17.64	0.00
309	9.98	8.00
310	13.52	8.00

TABLE 15. MODEL III, YEAR 2

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	28.03	11.00
102	11.70	8.00
103	14.53	11.00
104	24.09	6.00
105	24.60	6.00
106	22.81	6.00
107	39.18	11.00
108	14.95	8.00
109	31.21	3.00
110	17.01	11.00
111	18.72	6.00
112	25.44	6.00
113	27.00	22.00
114	22.08	11.00
115	13.90	3.00
116	9.35	0.00
117	19.03	8.00
118	14.38	3.00
119	54.89	19.00
120	23.42	6.00
121	40.91	35.00
122	69.46	36.00
123	15.50	0.00
124	10.36	3.00
125	19.65	6.00
126	21.30	14.00
127	29.22	9.00
128	39.22	27.00
129	37.53	27.00
130	32.07	17.00
131	3.67	0.00
201	30.79	0.00
202	8.47	16.00
203	10.54	8.00
204	0.00	0.00
205	42.68	11.00
301	16.20	8.00
302	1.49	0.00
303	8.94	8.00
304	17.83	0.00
305	10.28	3.00
306	8.75	8.00
307	12.78	8.00
308	18.78	16.00
309	13.10	3.00
310	13.14	8.00

TABLE 16. MODEL III, YEAR 3

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	24.04	14.00
102	8.78	8.00
103	16.82	14.00
104	26.84	6.00
105	24.17	3.00
106	21.30	16.00
107	32.93	6.00
108	17.34	6.00
109	18.78	3.00
110	12.38	16.00
111	21.48	9.00
112	27.95	14.00
113	18.07	6.00
114	25.38	11.00
115	9.30	3.00
116	5.92	3.00
117	30.57	6.00
118	16.98	6.00
119	62.53	22.00
120	21.93	6.00
121	77.54	51.00
122	105.39	49.00
123	17.92	3.00
124	6.29	0.00
125	26.82	6.00
126	16.60	6.00
127	30.42	11.00
128	44.85	14.00
129	57.49	25.00
130	24.65	19.00
131	8.46	8.00
201	28.57	0.00
202	8.81	16.00
203	8.86	8.00
204	16.14	8.00
205	46.44	0.00
301	13.31	0.00
302	8.03	11.00
303	15.27	3.00
304	12.00	16.00
305	7.75	8.00
306	11.11	8.00
307	12.44	16.00
308	15.52	3.00
309	7.86	0.00
310	13.89	8.00

TABLE 17. MODEL III, YEAR 4

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	27.26	14.00
102	13.30	8.00
103	31.82	6.00
104	22.85	14.00
105	16.44	8.00
106	21.47	6.00
107	24.50	19.00
108	24.71	3.00
109	30.16	19.00
110	23.44	11.00
111	25.07	3.00
112	23.13	6.00
113	36.06	9.00
114	23.05	14.00
115	15.15	11.00
116	5.64	3.00
117	14.87	3.00
118	14.64	6.00
119	55.89	11.00
120	16.68	6.00
121	86.31	48.00
122	109.82	36.00
123	21.61	3.00
124	17.27	3.00
125	25.03	19.00
126	24.83	3.00
127	23.06	19.00
128	56.06	19.00
129	49.40	20.00
130	33.77	27.00
131	13.45	3.00
201	28.60	0.00
202	10.85	8.00
203	24.73	0.00
204	9.53	8.00
205	8.38	8.00
301	10.09	0.00
302	10.99	3.00
303	10.01	8.00
304	13.71	16.00
305	12.77	0.00
306	10.32	8.00
307	11.36	8.00
308	16.59	3.00
309	10.88	8.00
310	17.20	16.00

TABLE 18. MODEL III, YEAR 5

ITEM NUMBER	KEEPING COSTS	ORDERING COSTS
101	32.96	11.00
102	13.65	11.00
103	24.32	11.00
104	27.82	6.00
105	15.75	11.00
106	16.52	11.00
107	35.04	8.00
108	18.10	3.00
109	21.20	16.00
110	14.85	11.00
111	21.34	6.00
112	15.13	11.00
113	30.33	19.00
114	38.24	6.00
115	19.97	0.00
116	10.74	0.00
117	29.86	3.00
119	53.88	19.00
118	20.27	3.00
120	22.80	6.00
121	61.14	38.00
122	95.98	41.00
123	28.24	3.00
124	17.99	0.00
125	27.19	9.00
126	15.66	6.00
127	25.15	14.00
128	60.93	17.00
129	53.28	20.00
130	29.30	22.00
131	21.03	3.00
201	11.85	0.00
202	11.00	16.00
203	16.09	16.00
204	23.39	8.00
205	39.70	16.00
301	9.55	0.00
302	7.15	8.00
303	14.01	8.00
304	13.40	3.00
305	9.90	8.00
306	7.25	8.00
307	11.57	8.00
308	22.49	16.00
309	15.71	8.00
310	12.68	11.00

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